### Innovation Program:
**Energy**

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### Project time line:
09.2010 – 08.2012

### Research project:
*A large-eddy simulation framework for wind energy studies*

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**Abstract**

The goal of this project is to develop a numerical modeling framework for aerodynamic simulations of wind turbines operating in the atmospheric boundary layer (ABL). In this framework, two major numerical techniques are adopted and combined: (a) a large-eddy simulation (LES) technique used to simulate turbulent air flows, and (b) a wind-turbine model used to parameterize the effects of wind turbines on the flows. In order to ensure the accuracy of this numerical modeling framework, experimental measurements collected in the wake of three-blade miniature wind turbines placed in a wind-tunnel boundary layer flow are used for model validation. The characteristics of the simulated turbine wakes (average velocity and turbulence intensity distributions) are similar to what we observed from the wind-tunnel measurements. In general, the numerical models can produce reasonable results in the turbine wake. Future efforts will focus on further development, validation and application of this framework in a variety of cases involving different atmospheric stability conditions (neutral, stable and unstable), land-surface characteristics (land cover and topography) and wind-farm layouts.

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**Schematic of the helicoidal vortex behind a wind turbine**

**Plots of the direction of air flows at different downwind cross sections obtained from different wind-turbine models. Color lines represent flow path lines in the turbine wake region.**
A large-eddy simulation framework for wind energy studies

1. Introduction

Accurate prediction of atmospheric boundary layer (ABL) flow and its interactions with wind turbines and wind farms is of great importance for optimizing the design (turbine siting) of wind energy projects. In particular, it can be used to maximize wind energy production and minimize fatigue loads associated with wind turbine wakes [1]. Additionally, numerical simulations can provide valuable quantitative insight into the potential impacts of wind farms on local meteorology [2]. These are associated with the significant role of wind turbines in slowing down the wind and enhancing vertical mixing of momentum, heat, moisture and other scalars.

Large-eddy simulation (LES) is the the state-of-the-art numerical technique used to study turbulent flows such as the ABL. LES can potentially provide the kind of high-resolution spatial and temporal information needed to maximize wind energy production and minimize fatigue loads in wind farms. However, the accuracy of LES in simulations of ABL flow with wind turbines hinges on our ability to parameterize subgrid-scale (SGS) turbulent fluxes as well as turbine-induced forces. Only recently have there been some efforts to apply LES to simulate wind-turbine wakes [3, 4].

In the proposed project, we introduce a new LES framework to study ABL flow and its interactions with wind turbines and wind farms. The SGS turbulent fluxes of momentum and heat are modeled using scale-dependent Lagrangian dynamic models [5], which optimize the local values of the SGS model coefficients and account for their scale dependence in a dynamic manner (using information of the resolved field and, thus, not requiring any tuning of parameters). The turbine-induced forces are parameterized with three wind turbine models: a standard actuator disk model without rotation (ADM-NR) that computes an overall thrust force and distributes it uniformly over the rotor disk; an actuator disk model with rotation (ADM-R) that computes the local lift and drag forces (based on blade element momentum theory) and distributes them over the rotor disk; and an actuator line model (ALM) that distributes those forces along lines that follow the position of the blades. In order to evaluate the performance of the proposed new LES framework (with scale-dependent Lagrangian dynamic models and different turbine parameterizations), simulation results are compared with high-resolution wind-tunnel measurements collected in the wake of a miniature wind turbine placed in a boundary layer flow.

2. LES framework

LES solves the filtered continuity equation, the filtered Navier-Stokes equations (written here in rotational form and using the Boussinesq approximation), and the filtered heat equation:
\[
\frac{\partial \tilde{u}_i}{\partial t} + \tilde{u}_j \left( \frac{\partial \tilde{u}_i}{\partial x_j} - \frac{\partial \tilde{u}_j}{\partial x_i} \right) = - \frac{1}{\rho} \frac{\partial \tilde{p}^*}{\partial x_i} - \nu \frac{\partial^2 \tilde{u}_i}{\partial x_j^2} + \delta_{ij} g \frac{\partial \tilde{\theta}}{\partial x_j} + f_i \epsilon_{ijk} \tilde{u}_k - f_i + F_i
\] (2)

\[
\frac{\partial \tilde{\theta}}{\partial t} + \tilde{u}_j \frac{\partial \tilde{\theta}}{\partial x_j} = - \frac{\partial q_j}{\partial x_j} + \alpha \frac{\partial^2 \tilde{\theta}}{\partial x_j^2}
\] (3)

where the tilde represents a three-dimensional spatial filtering operation at scale \( \Delta \), \( \tilde{u}_i \) is the resolved velocity in the \( i \)-direction (with \( i = 1, 2, 3 \) corresponding to the streamwise (x), spanwise (y) and vertical (z) directions), \( \tilde{\theta} \) is the resolved potential temperature, \( \theta_0 \) is the reference temperature, the angle brackets represent a horizontal average, \( g \) is the gravitational acceleration, \( f_i \) is the Coriolis parameter, \( \delta_{ij} \) is the Kronecker delta, \( \epsilon_{ijk} \) is the alternating unit tensor, \( \tilde{p}^* = \tilde{p} + \rho \tilde{u}_j \tilde{u}_j / 2 \) is the modified pressure, \( \tilde{p} \) is the filtered pressure, \( \rho \) is the air density, \( \nu \) is the kinematic viscosity of air, \( \alpha \) is the thermal diffusivity of air, \( f_i \) is an immersed force (per unit volume) for modeling the effect of wind turbines on the flow, and \( F_i \) is a forcing term (e.g., geostrophic wind or imposed mean pressure gradient). \( \tau_j \) and \( q_j \) are the SGS fluxes of momentum and heat, respectively, and are defined as

\[
\tau_j = \tilde{u}_j \mu_j - \tilde{u}_j \tilde{u}_j
\] (4)

and

\[
q_j = \tilde{u}_j \theta - \tilde{u}_j \tilde{\theta}
\] (5)

These SGS fluxes are unknown in a simulation and, therefore, they need to be parameterized. The detail information on scale-dependent Lagrangian dynamic models can be found in Stoll and Porté-Agel [5]. The parameterization of the turbine-induced forces can be found in Vermeer et. al. [1], Sorensen and Shen [7] and Wu and Porté-Agel [9].
3. Model Validation

To validate the new LES framework, simulation results are compared with the high-resolution velocity measurements [6] collected in the wake of a three-blade miniature wind turbine placed in the Saint Anthony Falls Laboratory atmospheric boundary-layer wind tunnel. The spatial distribution of two key turbulence statistics is used to characterize wind-turbine wakes: the time-averaged streamwise velocity $\bar{u}$, and the streamwise turbulence intensity $\sigma_u/\bar{u}_{hub}$. Figures 1 and 2 show contours of the time-averaged streamwise velocity as well as the streamwise turbulence intensity, respectively, obtained from the wind-tunnel experiment and simulations with ADM-NR, ADM-R and ALM on a vertical plane perpendicular to the rotor area. Furthermore, to facilitate the quantitative comparison of the results, Figures 3 and 4 show vertical profiles of the time-averaged streamwise velocity as well as the streamwise turbulence intensity, respectively, at selected downwind locations ($x/d=-1, 2, 3, 5, 7, 10, 14, 20$, where $d$ is the rotor diameter). As shown in Figures 1 and 3, LES with the ALM or the
ADM-R yields mean velocity profiles that are in good agreement with the measurements everywhere in the turbine wake (near wake as well as far wake). The ADM-NR is able to capture the velocity distribution in the far-wake region ($x/d>5$), but it clearly overpredicts the velocity in the center of the wake in the near-wake region ($x/d<5$). This failure of the ADM-NR to reproduce the velocity magnitude in the near-wake region can be attributed to the limitations of two important assumptions made in the ADM-NR: (a) the effect of turbine-induced rotation is ignored, and (b) the force is uniformly distributed over the rotor disk, thus ignoring the radial variation of the force. These two assumptions are in contrast with simulation results of the non-uniform force distribution reported by Sørensen and Shen [7].

Figure 2: Contours of the streamwise turbulence intensity $\sigma_u/u_{hub}$ on the middle vertical plane perpendicular to the turbine: (a) wind-tunnel measurements, (b) ADM-NR, (c) ADM-R, (d) ALM.
In Figure 2, as reported by Chamorro and Porté-Agel [8, 6], the measurements show a

Figure 3: Comparison of vertical profiles of the time-averaged streamwise velocity $\overline{u}$ (m/s): wind-tunnel measurements (o), ADM-NR (dashed line), ADM-R (solid line) and ALM (dotted line).

Figure 4: Comparison of vertical profiles of the streamwise turbulence intensity $\sigma_u/\overline{u}_{hub}$: wind-tunnel measurements (o), ADM-NR (dashed line), ADM-R (solid line) and ALM (dotted line).
Strong enhancement of the turbulence intensity (compared with the relatively low turbulence levels in the incoming flow) at the level of the top tip. This turbulence intensity distribution and, in particular, the maximum enhancement of turbulence intensity occurring at the top-tip level can be explained considering the non-axisymmetric distribution of velocity profiles (Figures 1 and 3) and the fact that the mean shear and associated turbulence kinetic energy production are maximum at the top-tip height. Simulations using the three wind-turbine models yield similar qualitative trends in the turbulence intensity distribution and the location of the maximum value. However, all models differ in their ability to capture the magnitude of the turbulence intensity. In particular, the turbulence intensity profiles obtained with both ALM and ADM-R are good agreement with the wind-tunnel measurements. The ADM-NR clearly underestimates the wake turbulence intensity at the downstream positions of x/d=2, 3 and 5, and therefore its maximum value. However, like in the case of the mean velocity, all wind-turbine models give a good prediction in the far-wake region, particularly after 10 rotor diameters.

4. Summary

The proposed LES framework is validated against high-resolution velocity measurements collected in the wake of a miniature wind turbine placed in a wind-tunnel boundary layer flow. In general, the characteristics of the simulated turbine wakes (average velocity and turbulence intensity distributions) are in good agreement with the measurements. The comparison with the wind-tunnel measurements shows that the turbulence statistics obtained with LES and the ADM-NR have some differences with respect to the measurements in the near-wake region. In particular, the model overestimates the average velocity in the center of the wake, while underestimating the turbulence intensity at the top-tip level, where turbulence levels are highest due to the presence of a strong shear layer. The ADM-R and ALM yield more accurate predictions of the different turbulence statistics in the near-wake region. This highlights the importance of having a wind-turbine model that allows for non-uniform distribution of the turbine-induced forces. In the far wake, all three models produce reasonable results.

5. References


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